# Midterm 2 - Booklet 1

## Eco 231 - Undergraduate Econometrics

11/08/2010 (Prof. Carolina Caetano)

### INSTRUCTIONS

Reading and understanding the instructions is your responsibility. Failure to comply may result in loss of points, and there will be no leniency on that respect.

1. You have received two booklets. This booklets contains the exam instructions and the exam questions. The second booklet contains the numbered pages where you will answer the questions. You must sign both booklets in the space provided.
2. This exam has 3 questions and it is worth 100 points. You have until 5 minutes before the end of the regular class time to answer it.
3. You must answer each question in the Booklet 2, exactly in the space provided for each question. You may use the back of the pages if they are empty. If you answer a question out of the order, or otherwise not on the space provided for it in the second booklet, your question will not be graded. If you need more space, you must ask for extra paper from the TA. It is your responsibility at the end of the exam to staple the extra page exactly in the right place in your exam. You may ask for draft paper if you like.
4. You are not allowed the use of notes, cheat sheets, calculators, or electronic devices of any kind. If your answers are unclear or illegible you may lose points. You may answer in pencil.
5. You may return the exam at any time until 10 minutes before the end of class time. If you finished within 10 minutes of the end of class time, you must remain seated. Do not get up when the TA announces the time is up. Remain seated and follow the TA's instructions.
6. At the end of the exam, you must hand both booklets together.
7. Sign and print your name below. You signature demonstrates that you have read and understood the instructions. An exam without the signature will not be graded.

1. **Name (print):** _____
2. **Class time:** _____
3. **Signature:** _____

**Question 1 (30 points):**

**1.a)** (10 points) Enunciate the Gauss-Markov Theorem. You must write all the assumptions required (not only their names), and then state the theorem.

**1.b)** (5 points) Why is the Gauss-Markov theorem important?

**1.c)** (5 points, hard) Suppose that

$$y = \beta_0 + \beta_1 x_i + u$$

satisfies the Gauss-Markov assumptions. Let

$$\tilde{\beta}_1 = \frac{\sum_{i=1}^{n} w_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} w_i (x_i - \bar{x})^2}.$$

This is in fact an unbiased estimator of $\beta_1$. Compare the variance of this estimator to the variance of the OLS estimator

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}.$$

and explain your reasoning.

**1.d)** (5 points) Suppose that by coincidence, when you plugged the sample numbers, $\hat{\beta}_1 = \tilde{\beta}_1$. You would like to test the hypothesis

$$H_0 : \beta_1 = 0$$

against the alternative hypothesis

$$H_1 : \beta_1 \neq 0$$

and it turns out that if $\beta_1 = 0$,

$$\mathbb{P}\,() = 0.05.$$

Using $\tilde{\beta}_1$, when would you reject $H_0$ if you want your test to be of 5% significance level?

**1.e)** (5 points) Suppose that you followed your own rule, but when you used the test based on $\tilde{\beta}_1$ you did not reject $H_0$. Would you reject $H_0$ using the test based on $\hat{\beta}_1$ that we learned in class?

**Question 2 (40 points):** Are rents influenced by the student population in a college town? Let *rent* be the average monthly rent paid on rental unites in a college town in the

United States. Let *pop* denote the total city population, *avginc* the average city income, and *pctstu* the student population as a percentage of the total population. One model to test for a relationship is:

$$\log(rent) = \beta_0 + \beta_1 \log(pop) + \beta_2 \log(avinc) + \beta_3 pcpstu + u.$$

Consider the dual sided critical values of the Student-$t$ distribution:

| degrees of freedom | critical values 10% | critical values 5% | critical values 1% |
|---|---|---|---|
| 60 | 1.67 | 2.0 | 2.66 |
| 61 | 1.67 | 2.0 | 2.66 |
| 62 | 1.67 | 2.0 | 2.66 |
| 63 | 1.67 | 2.0 | 2.66 |
| 64 | 1.67 | 2.0 | 2.65 |

and the critical values of the F distribution for the 5% significance level test:

| | | Degrees of freedom: Numerator | | | |
|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** |
| Degrees of freedom: Denominator | **60** | 4.0 | 3.15 | 2.76 | 2.52 |
| | **61** | 4.0 | 3.15 | 2.75 | 2.52 |
| | **62** | 4.0 | 3.14 | 2.75 | 2.52 |
| | **63** | 3.99 | 3.14 | 2.75 | 2.52 |
| | **64** | 3.99 | 3.14 | 2.75 | 2.51 |

**2.a)** (10 points) How would you test (with a significance level of 1%) the hypothesis that an increase of 10% of the size of the student body relative to the population has *ceteris paribus* the same effect on monthly rent as an increase of 1% in the population?

**2.b)** (10 points) How would you test (with a significance level of 5%) the hypothesis that a 1% increase in income has the same effect in the rents as a 10% change in the population, while *ceteris paribus* the student population has no effect at all?

**2.c)** (10 points) Suppose that someone estimated this model and reported the findings in the following table:

| Explanatory Variables | Regression Results |
|---|---|
| log(*pop*) | 0.654 |
| | (0.039) |
| log(*avging*) | 0.507 |
| | (0.081) |
| *pctstu* | 0.0056 |
| | (0.0017) |
| intercept | 0.043 |
| | (0.844) |
| Observations | 64 |
| R-squared | 0.458 |

Write and defend your opinion about the statement: if the population increases 10%, then *ceteris paribus* the rents will increase 6.5%.

**2.d)** (10 points, hard) How might your opinion change if we excluded the variable *pctstu* from the model?

**Question 3 (30 points):** Suppose that we are studying fertility decisions in the population, and we have a sample of $n$ women. We know if they are currently single (never married), married, divorced or widowed, and we also know their fertility (number of children). In the following questions, you may be tempted to answer directly. Instead, always show the math, so that I can be sure you understand what you are doing.

**3.a)** (5 points) Write a simple model of fertility relating the number of children a woman has to her marital status.

**3.b)** (5 points) How would you estimate the difference in fertility between married and divorced women of the same age, using as base category the single women?

**3.c)** (5 points) How do you interpret $\beta_0$ in this model?

**3.d)** (5 points) How would you interpret your findings in (2b) if the base category changed to widowed?

**3.e)** (5 points) How would you interpret $\beta_0$ if the base category changed to widowed? Is $\beta_0$ larger or smaller than in (2c)?

**3.f)** (5 points, hard) Suppose that you also have data on the women's age. How would you estimate (using an OLS model) the difference in fertility between married and divorced women that are 50 years of age or older?

# Booklet 2 - Midterm 2 (11/08/10) - Eco 231W

1. **Name:** _____

2. **Student ID:** _____

3. **Class time:** _____

## Good wishes!

**1.a) (10 points)**

**1.b) (5 points)**

**1.c) (5 points)**

**1.d) (5 points)**

**1.e) (5 points)**

**2.a) (10 points)**

**2.b) (10 points)**

**2.c) (10 points)**

**2.d) (10 points)**

**3.a) (5 points)**

**3.b) (5 points)**

**3.c) (5 points)**

**3.d) (5 points)**

**3.e) (5 points)**

**3.f) (5 points)**